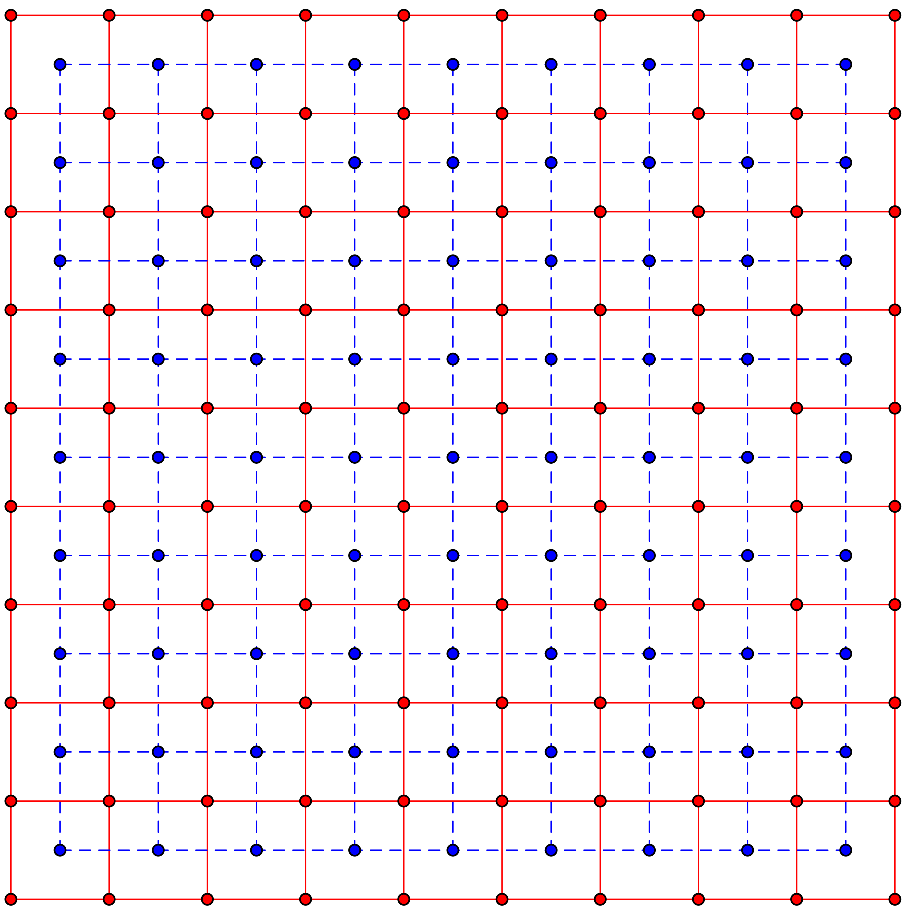
Dissection Puzzles and the Geometry of Tessellations

It can be proved that tessellations with repeating patterns follow stringent rules. The "repeats" are transformations that are symmetries, so the composition of these transformations also must be symmetries. This restricts the patterns of these symmetries.

As an example, let's consider tessellations that have 90-degree rotations among the symmetries. We observed that the square tessellation had a pattern of two kinds of 90-degree centers. One set of centers was made up of centers of the squares. The other set was the set of vertices of the squares. Finally, the midpoints of the sides of the squares are the centers of 180-degree rotations.

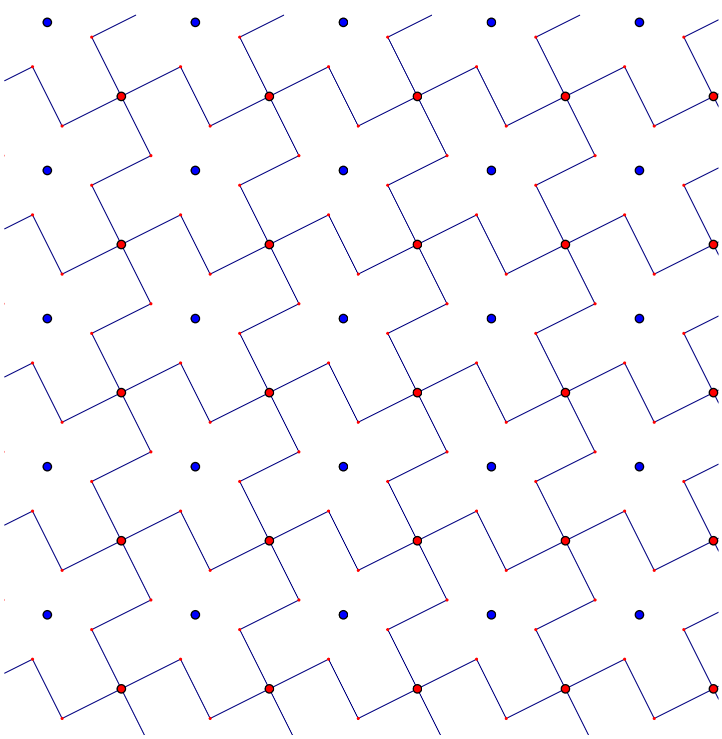
Thus the pattern looks like this: one set of (red) centers forms a square pattern and the other (blue) set is the set of centers of these squares, also forming a square patterns.



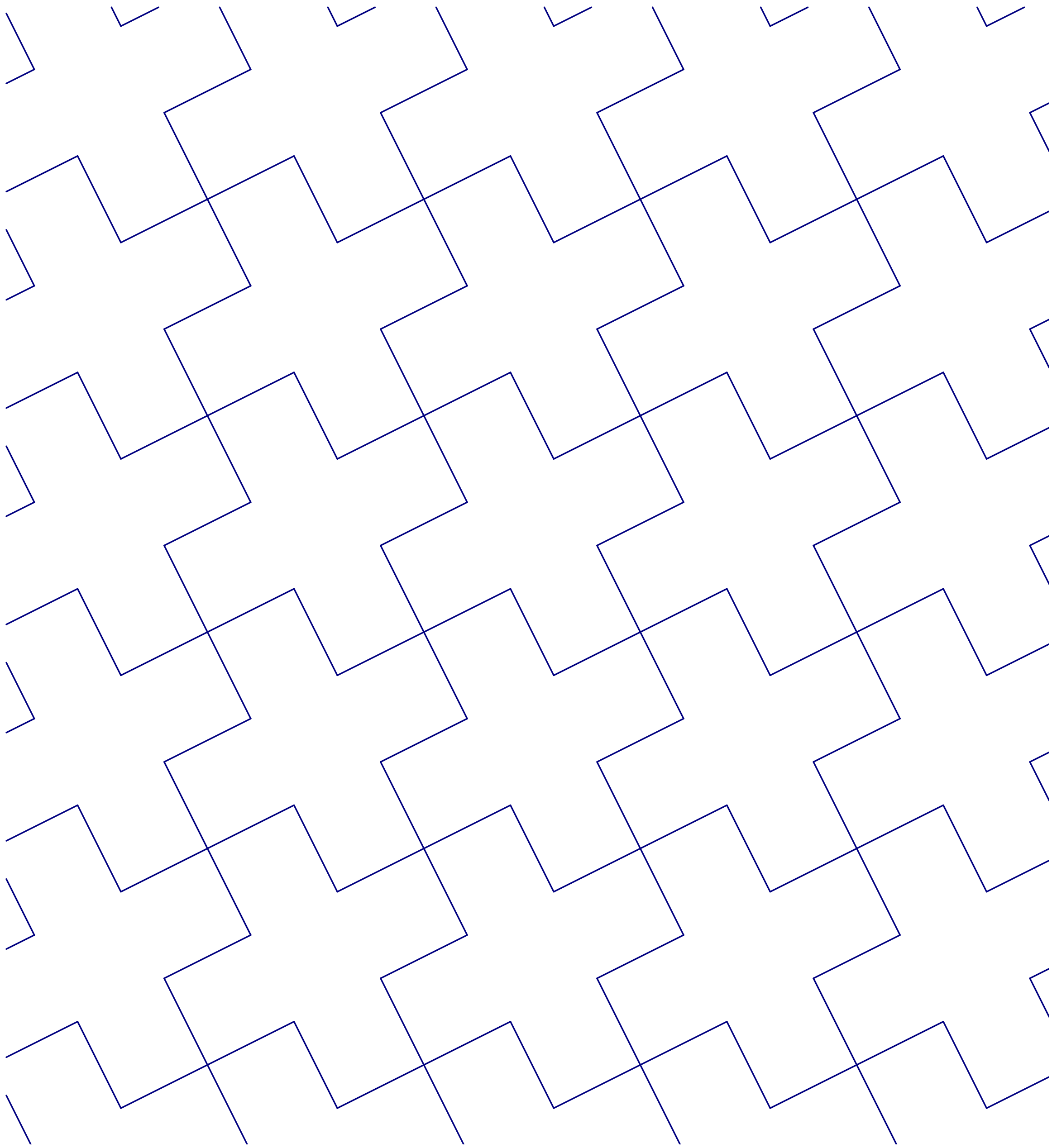
In addition to the rotations, a key feature is that the translation symmetries are the translations that take a red center to a red center (these automatically take blue centers to blue centers, etc). Thus one red square is a **fundamental region** for the pattern. Such a square is translated by symmetries to cover the whole plane.

Look for the square center pattern in this Escher design:

Then see it in this tessellation by crosses:

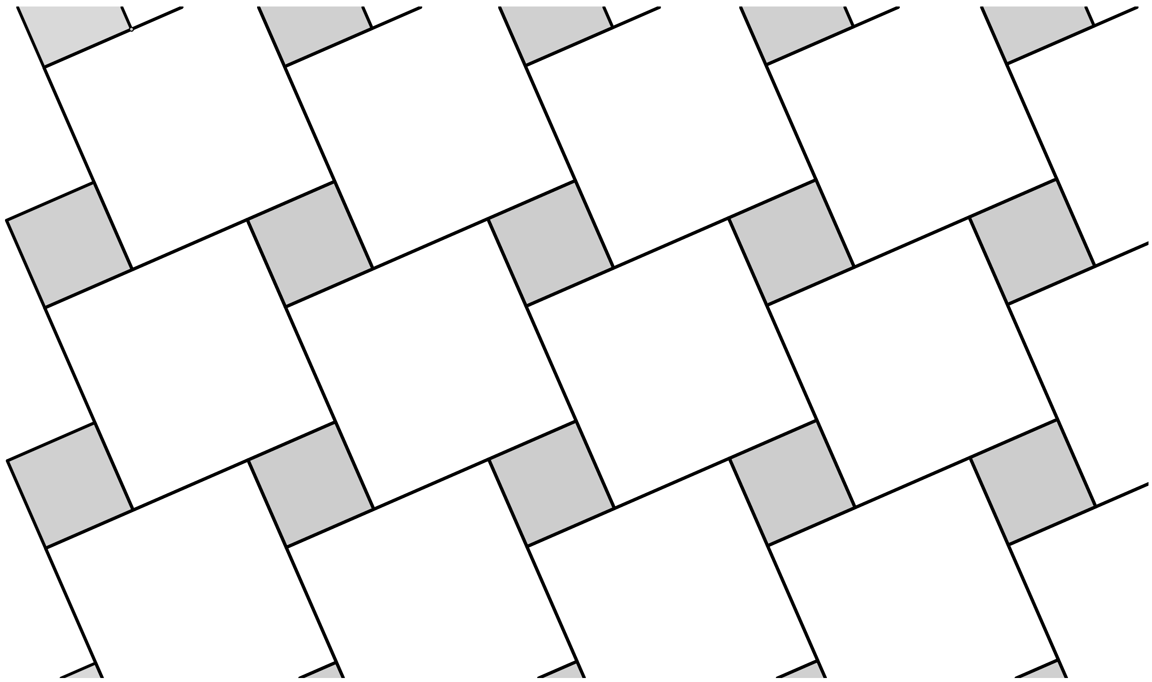


Making a Puzzle: In this pattern, if one picks four points that correspond by a translation symmetry so that the points are the vertices of a square. Then cut out the square and cut along the lines of the design. Then the pieces can be reassembled either as a square or a cross.



NOTE: The AREA of the square is equal to the AREA of the cross.

Now one more tessellation that you may have seen on floors (one is in the HUB food area).



Again, find a square so that the two squares can be reassembled as a square.